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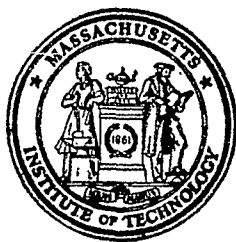
DEPARTMENT OF CIVIL AND SANITARY ENGINEERING

TECHNICAL REPORT No. 10

RESISTANCE COEFFICIENTS

FOR

ACCELERATED FLOW THROUGH ORIFICES

BY

JAMES W. DAILY AND WILBUR L. HANKEY, JR.

OCTOBER 1953

HYDRODYNAMICS LABORATORY

Department of Civil and Sanitary Engineering

Massachusetts Institute of Technology

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ABSTRACT

The investigation of fluid friction in unsteady motion conducted in the M.I.T. Hydrodynamics Laboratory was extended to include frictional resistance caused by orifices in accelerated flow. The objective of this program was to measure the frictional resistance under accelerated conditions along a circular conduit containing an orifice. Two different orifices were investigated, each located in a region of fully developed conduit boundary layer and velocity profile. The results are compared with previous measurements of the resistance of a clear smooth conduit.

The orifices of area ratios 0.5 and 0.7 were installed 38-1/2 diameters from the entrance nozzle in the one-inch diameter tube which forms the working section of the Unsteady Flow Water Tunnel. The velocity and acceleration of flow was programmed by a servomechanism which controls the pressure difference between the supply and receiving tanks of the tunnel. The instantaneous head drop across a foot length of tube containing an orifice was measured with diaphragm differential transformer type differential pressure cells. The velocity head was measured with a similar cell across the metering entrance nozzle and acceleration computed from time incremental changes in the velocity. The tests with orifices covered a conduit Reynolds number range from 5×10^4 to 3×10^5 and accelerations up to 35 ft/sec^2.

The important conclusions from the orifice tests are:

1. The coefficient of head drop K_a is independent of Reynolds number and is a function of the acceleration parameter $\frac{aL}{V^2}$.

2. The frictional resistance for a given instantaneous velocity of accelerated flow through an orifice in a tube is appreciably less than for steady flow at the same velocity.

3. The frictional resistance for a given instantaneous velocity of accelerated flow through an orifice in a tube decreases with increasing acceleration.

A reanalysis of the previous measurements with a clear conduit shows:

4. The coefficient of head drop K_a is a function of Reynolds number as well as $\frac{aL}{V^2}$.

5. The frictional resistance for a given instantaneous velocity of accelerated flow through a uniform diameter smooth tube is equal to, or possibly slightly greater than, that for steady flow at the same velocity.

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I INTRODUCTION

A. Background

The previously reported measurements (Refs. 1 and 2) of fluid friction for accelerated flow in a circular conduit were made using a one-inch diameter "working section" in the Unsteady Flow Water Tunnel (Ref. 3). This tube has a length of 99 inches (99 diameters) and the tests were performed in a section in which the boundary layer and velocity profile were fully developed for steady flow. The fluid was accelerated from one steady velocity to another with accelerations up to 35 fps^2 between Reynolds numbers from about 7.5×10^4 to 7.5×10^5. The results showed no appreciable change in the friction factor between steady and unsteady conditions.

B. Objective of Current Experiments

The above measurements have been followed by the investigation of another form of fluid friction loss, that associated with sudden transitions (Ref. 4). The study was concerned with the effect of unsteady motion on the dissipation of energy associated with high shear and turbulence generation accompanying separation and jet formation. Since an orifice plate could be readily installed in the existing tunnel, the investigations described below have been made with sharp-edged orifices of different diameters. The orifices investigated were located in a region along the one-inch diameter tube where the boundary layer and velocity profile were fully developed under conditions of steady flow.

II THEORY

A. General

Considerable information is available on orifice head losses in steady motion. Very little is known pertaining to the evaluation of unsteady orifice losses; however, information has been obtained for the drag on a moving disk under accelerated conditions (Ref. 5). The results relate the unsteady coefficient of drag to the Reynolds number and a correlating modulus, $\frac{ad}{V^2}$ (a = acceleration, d = disk diameter, V = velocity). The force necessary to tow an immersed object is composed of a compound drag term, made up of skin friction drag, a turbulent wake form drag and an accelerative force, comprising the mass of the object and the associated virtual mass. The force necessary to accelerate a fluid through an orifice is composed of a compound frictional term, made up of skin friction and a turbulent wake force, and an accelerative force. By comparison, an analogy between the drag coefficient of an immersed disk and the head loss coefficient for an orifice may be formulated. The following analysis is the derivation of a similar correlating modulus for orifices.

B. Unsteady Flow Equations

The following momentum analysis of the unsteady flow of an incompressible fluid through an orifice in a uniform diameter pipe assumes the motion to be one-dimensional. At any instant only variations in a longitudinal direction are

considered, average values of velocity pressure and acceleration being assumed to hold over any normal cross section. Random turbulent fluctuations are not introduced explicitly, but their effects, if any, are absorbed into the overall resistance coefficients employed. For steady motion, the analysis reduces to the conventional energy equation with the resistance coefficients as measures of energy loss. It will be assumed that the differences between coefficients for steady and unsteady motion is a true measure of the transient effects although it is recognized not only that corrections should be made for deviations from one-dimensional flow, but that resistance coefficients from a force-momentum equation may not be equal to the loss coefficients from an energy analysis. (Ref. 6)

The continuity equation retains its usual form,

$$\frac{\partial (Au)}{\partial x} = 0 \qquad \qquad \text{(Eq. 1)}$$

where A, u, and x are defined in the accompanying definition sketch and table of symbols.

Definition Sketch with Notation
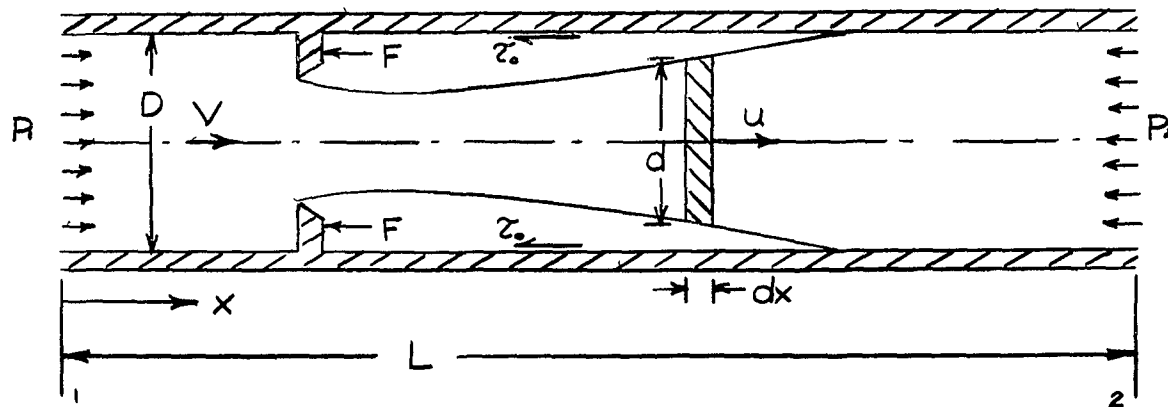
A = cross-sectional flow area

d = jet diameter at any x

D = conduit diameter

F = orifice drag force

L = conduit test length

m = orifice diameter ratio

p = pressure intensity

t = time

u = instantaneous cross-sectional mean velocity of jet at any x

V = instantaneous cross-sectional mean velocity in conduit

x = distance in flow direction

ρ = fluid density

τ_o = wall shear

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The equation of motion is derived by considering the equilibrium of forces on a differential fluid element. The rate of change of momentum ($\rho A \frac{du}{dt} dx$) is equated to the sum of the normal end forces ($-A \frac{\partial p}{\partial x} dx$), the "x" component of the wall shear ($-\tau_o \pi D dx$) and the portion of the orifice drag force in a length, L, effective on the differential element ($-F \frac{dx}{L}$).

$$\rho A \frac{du}{dt} dx = -A \frac{\partial p}{\partial x} dx - \tau_o \pi D dx - F \frac{dx}{L} \qquad \text{(Eq. 2)}$$

Expanding and substituting $\tau_o \equiv c_f \rho \frac{u^2}{2}$ $\qquad$ (Eq. 3)

$$F \equiv KA\rho \frac{V^2}{2} \qquad \text{(Eq. 4)}$$

where c_f = coefficient of local wall friction

$\qquad$ K = orifice drag coefficient

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = - \frac{\partial p}{\partial x} - 4c_f \frac{D}{d^2} \rho \frac{u^2}{2} - \frac{K}{L} \rho \frac{V^2}{2} \qquad \text{(Eq. 5)}$$

Integrating with respect to "x" between points 1 and 2, a length L, such that $u_1 = u_2 = V$, will yield the following equation:

$$p_1 - p_2 = \rho \int_o^L \frac{\partial}{\partial t} u\, dx \;+\; 2\rho D \int_o^L \frac{c_f}{d^2} u^2 dx + K\rho \frac{V^2}{2} \qquad \text{(Eq. 6)}$$

The integral term for the wall friction can be replaced in terms of a mean coefficient so that

$$p_1 - p_2 = \rho \int_o^L \frac{\partial u}{\partial t} dx + K_f \rho \frac{V^2}{2} + K \rho \frac{V^2}{2}$$

or in dimensionless form

$$\frac{p_1 - p_2}{\rho \frac{V^2}{2}} = \frac{2}{V^2} \int_o^L \frac{\partial u}{\partial t} dx + K_f + K \qquad \text{(Eq. 7)}$$

In order to compare the results of steady and unsteady flow measurements and thus evaluate the effects of unsteadiness, it is convenient to reduce Eq. 7 to another form. First, for this particular problem, let us assume that the ratio $\frac{u}{V}$

for any point along the stream is dependent on x only so that we may write the inertial term in Eq. 7 as

$$\rho \int_0^L \frac{\partial u}{\partial t}\, dx = \rho \frac{\partial V}{\partial t} \int_0^L \frac{u}{V}\, dx$$

Next, we will introduce the following definitions and equalities:

1. K_a, coefficient of head drop in accelerated motion,

$$K_a = \frac{p_1 - p_2}{\rho \frac{V^2}{2}} \qquad \text{(Eq. 8)}$$

2. c_1, coefficient of inertial head drop,

$$c_1 = \frac{1.0}{L} \int_0^L \frac{u}{V}\, dx \qquad \text{(Eq. 9)}$$

c_1 can be evaluated from a flow net of the jet profile through the orifice and is a constant if the jet profile does not change with velocity or acceleration.

3. a, local and, in this case, also the total acceleration in the conduit away from the orifice plate,

$$a = \frac{\partial V}{\partial t} \qquad \text{(Eq. 10)}$$

4. K_s, coefficient of total resistance (wall and orifice) for steady flow at the instantaneous velocity; and K_t, correcting coefficient which gives a measure of the additional transient effects such that

$$K_s + K_t = K_f + K \qquad \text{(Eq. 11)}$$

where

$$K_s = \left. \frac{p_1 - p_2}{\rho \frac{V^2}{2}} \right|_{a = 0} \qquad \text{(Eq. 12)}$$

Eq. 11 is merely a restatement of the consideration that both K_f and K as defined by their use in Eq. 7 may include steady and unsteady components. In general, K_s is a function of Reynolds number, the orifice-to-conduit diameter ratio, and the absolute roughness of the boundaries.

For unsteady flow through a particular orifice, K_s in the equality given by Eq. 11 is taken as the steady state value corresponding to the

instantaneous Reynolds number of some transient condition. Presumably also, K_t should be a function of Reynolds number as well as an acceleration parameter and the geometric parameters of the test conduit.

Using the above definitions, Eq. 7 is reduced to the following coefficient form:

$$K_a = 2c_1 \frac{aL}{V^2} + K_s + K_t \qquad \text{(Eq. 13)}$$

in which $\frac{aL}{V^2}$ is an acceleration parameter.

Equation 13 can be further simplified with the aid of an analogy to Schonfeld's analysis for smooth round tubes (Ref. 7). Schonfeld presents the following solution for slowly varied motion in which the resistance dominates (as opposed to quickly varied motion where the inertia dominates).

$$p_1 - p_2 = \frac{\rho gL}{R_h C'^2} \frac{Q^2}{A^2} + N \frac{dQ}{dt} \qquad \text{(Eq. 14)}$$

where Q = rate of discharge

R_h = hydraulic radius

C' = steady flow Chezy coefficient

$$N = \frac{\rho L}{A} \left[1.0 + \frac{234}{(C' + 14.0)^2} \right] \qquad \text{(Eq. 15)}$$

By substituting $C' = \sqrt{\frac{8g}{f}}$ (f = steady flow friction factor), $R_h = \frac{D}{4}$ and $Q = AV$, Eq. 14 can be rewritten for the tunnel test section thus

$$\frac{p_1 - p_2}{\rho \frac{V^2}{2}} = \frac{2aL}{V^2} + f \frac{L}{D} + \frac{0.91}{\left(\frac{1}{\sqrt{f}} + 0.87\right)^2} \frac{2aL}{V^2} \qquad \text{(Eq. 16)}$$

Comparing with Eq. 13 we note that

$c_1 = 1.00$

$$K_s = \frac{fL}{D} \qquad \text{(Eq. 17)}$$

$$K_t = \frac{0.91}{\left(\frac{1}{\sqrt{f}} + 0.87\right)^2} \frac{2aL}{V^2} \qquad \text{(Eq. 18)}$$

The difference between the resistance for accelerated motion and that for steady motion takes the form of a correction for the inertial term.

Using an analogous treatment for orifice resistance, let

$$K_t = c_2 \frac{2aL}{V^2} \qquad \text{(Eq. 19)}$$

in which c_2 is a function of the orifice steady flow coefficient, K_s. Then Eq. 13 reduces to the following:

$$K_a = K_s + (c_1 + c_2) \frac{2aL}{V^2} \qquad \text{(Eq. 20)}$$

with

$$c = c_1 + c_2$$

We can also write this as

$$\frac{K_a}{K_s} = 1 + c \frac{2aL}{K_s V^2} \qquad \text{(Eq. 21)}$$

Recall now that c_1 is a function of the jet profile and hence for a given orifice area ratio and test length is probably dependent upon Reynolds number R and the acceleration parameter $\frac{aL}{V^2}$. Also, c_2 is assumed to be a function of K_s and hence dependent upon the Reynolds number and m^2. For one orifice in a velocity and acceleration region where the jet profile and K_s do not change greatly, c is a constant. Using Eq. 21, K_a can be determined for any $\frac{aL}{V^2}$ ratio if c and K_s are known.

For accelerated flow in the positive x-direction, K_s and K_a are both positive with K_a greater than K_s. Thus, c in Eq. 21 is a positive quantity. Of the two terms comprising c, c_1 is 1.00 for an orifice-to-conduit diameter ratio of 1.00 and increases with decreasing diameter ratio (Eq. 9) while c_2 is zero if acceleration does not affect the frictional resistance (Eq. 19). Using Eqs. 11 and 13 together with Eq. 19, the frictional resistance terms can be written as

$$(K_f + K) = K_a - c_1 \frac{2aL}{V^2}$$

$$= K_s + c_2 \frac{2aL}{V^2} \qquad \text{(Eq. 22)}$$

In this form it is clear that a negative c_2 will indicate less frictional resistance for accelerated than for steady motion and vice versa. The magnitude and sign of these coefficients remain to be experimentally evaluated for each orifice ratio.

III PROCEDURE

A. Type and Scope of Experiments

As indicated by the development in the previous section, the problem was to separate the inertial and frictional components of the "extra" total resistance measured with accelerated flow through a length of pipe containing an orifice.

The basic experimental measurements were total head drop versus velocity for various accelerations from zero upward. The same Reynolds number range (based on the instantaneous mean conduit velocity V) was covered by the steady and unsteady tests. With this data and with c_1 evaluated from a flow net, all the terms in Eqs. 7, 19 and 21 could be determined.

Two orifices of diameter ratios 0.707 and 0.837 (area ratios 0.50 and 0.70) were used in tests covering a conduit Reynolds number range from 5×10^4 to 3×10^5 and accelerations up to 35 ft/sec^2. This corresponded to a velocity range of about 0 - 40 feet per second and a minimum test duration (for acceleration runs) of about one second. In addition, coefficients for the limiting "orifice," of area ratio equal 1.0, were calculated from the results of the previous investigation of resistance in a uniform conduit (Ref. 1). These uniform conduit tests were made in the R range between $7.5 \times 10^4 \times 7.5 \times 10^5$.

It was desired to have as large a range of the modulus, $\frac{aL}{V^2}$, as possible; therefore it was decided to start from rest or some low steady velocity and accelerate. A range of the modulus, $\frac{aL}{V^2}$, within which an infinite number of values are available, may be obtained from one acceleration of the fluid from rest, thus necessitating only a few test runs.

B. Apparatus

1. Tunnel - The apparatus used for this experiment is a non-return, unsteady flow water tunnel. The same tunnel, control system and recording system were used in the fluid friction investigation of unsteady flow through conduits and are aptly described in Refs. 1, 2 and 3.

The schematic section of the tunnel is shown in Fig. 1. The tunnel consists of two cylindrical tanks mounted one above the other and connected by a vertical pipe or working section which contains the orifice. Water is caused to flow from one tank to the other under pneumatic control. Compressed air in the spaces above the water surfaces in the two tanks is used to provide an adequate driving force for a desired acceleration. To obtain the ranges of acceleration and pressure in the working section, compressed air must be admitted to or rejected from either tank according to some time schedule. A closed loop automatic control for velocity and acceleration is provided, which operates on the "error" between scheduled and actual tank pressure differences (Ref. 8). A cam-driven pressure-programming device drives a control valve, between the high-pressure reservoir and the top tank, through which critical flow is maintained. In order to avoid cavitation and prevent air from entering the piezometric system, the test section is maintained at positive pressure by manually throttling the exhaust from the bottom tank.

Diaphragm differential transformer type differential pressure cells are used as the pressure-sensing devices. An oscillator and preamplifier energize the cell's transformer. Each pressure gage signal is sent through a separate amplifying and detecting unit prior to being recorded on a 10-inch wide roll chart of Hathaway Type S8-C oscillograph.

Natural frequencies, measured with water-filled lead lines, pressure cells and the recording system have exceeded 165 cps.
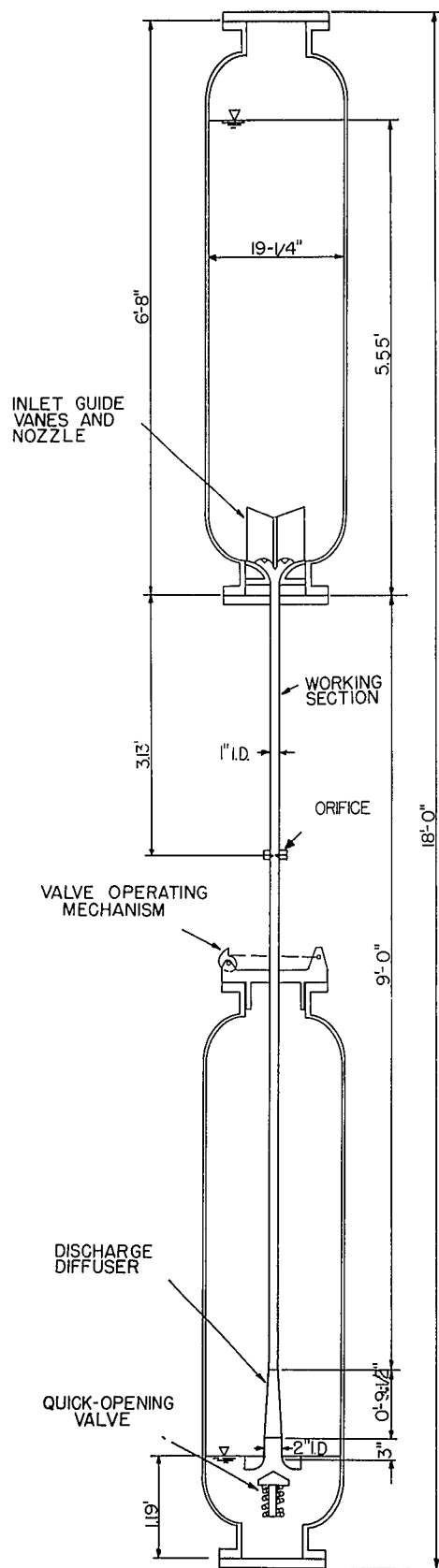
INLET GUIDE
VANES AND
NOZZLE

WORKING
SECTION

1" I.D.

ORIFICE

VALVE OPERATING
MECHANISM

DISCHARGE
DIFFUSER

QUICK-OPENING
VALVE

2" I.D.

19-1/4"

6'-8"

5.55'

3.13'

18'-0"

9'-0"

9'-0 3/4"

3"

1.9'

Fig. 1 Schematic Section of the Tunnel

2. <u>Test Section and Orifice</u> - A test section of 1-inch diameter brass tubing 99 inches long contains the orifice. The orifices used were square edged and were constructed of brass according to ASME Standard Specifications (Fig. 2). The orifice plate was enclosed in flanges with corner taps and provided with a vena contracta tap. The lower flange was soldered to the test section while the upper flange, provided with an "O" ring, was left movable to facilitate the exchange of orifice plates. The plate was positioned with dowels and the flanges secured with studs (Figs. 3 and 4).

It was desirable to have the orifice in a fully developed flow region in order to reduce the complexity of the problem. When fluid enters a conduit, a boundary layer develops and grows until the layers meet at the centerline. Beyond this point, the flow is fully developed and the velocity profile is unchanged downstream. For completely turbulent conditions at the conduit entrance, the initial point of fully developed flow, x_c, measured from the conduit inlet, can be determined approximately from the following equation (Ref. 9):

$$x_c = 0.7 \; D \; \left(\frac{V \; x_c}{\nu} \right)^{1/4}$$

For the case of the water tunnel conduit:

| V (fps)	0	10	20	30	40	80
x_c (in.)	0	11.9	14.1	15.6	16.8	20.0

A convenient location for the orifice existed at a distance of 38-1/2 inches from the entrance nozzle. Since the velocities encountered were between 0 and 30 fps, this location was considered to be in a fully developed flow region and was used for the orifice.

The test length L was between piezometer taps located 4-1/2D upstream and 7-1/2D downstream of the orifice plate. These locations bracketed the range of expected influence of the orifice on the local flow conditions. Fink and Pollis (Ref. 10) show longitudinal pressure profiles along a tube for orifice-diameter ratios between 0.3 and 0.7 and pipe Reynolds numbers between 10^4 and 10^5 which indicate that all non-uniform flow conditions are confined to the stretch between 1D upstream and 4D downstream of the orifice plate. In addition, from the previously noted findings (Ref. 2) that the frictional resistance in uniform flow is essentially independent of acceleration, it was inferred that the re-establishment of uniform flow downstream of the orifice should give the same degree of turbulence and rate of turbulent energy dissipation downstream as upstream.

C. <u>Test and Computational Procedure</u>

The general test procedure as described in detail in Refs. 1, 2 and 4 was used. The test data appeared as traces on the oscillograph chart of the differential pressure across the metering entrance nozzle and the pressure drop along the conduit containing the orifice versus time. By referring to static calibrations performed before and after each test run, the instantaneous differential
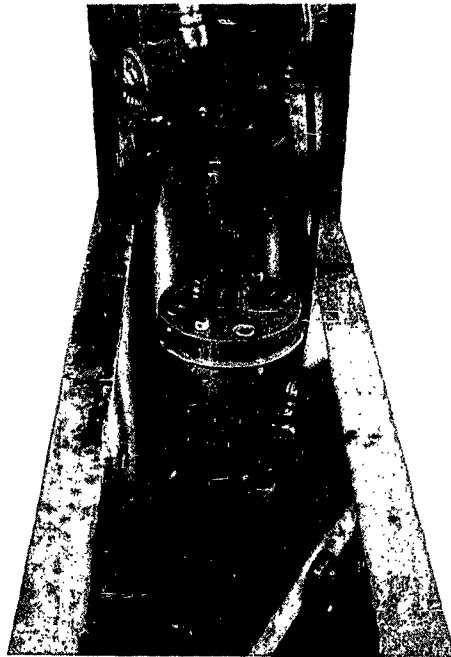
Fig. 2 Detailed Drawing of the Orifice
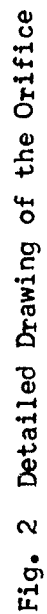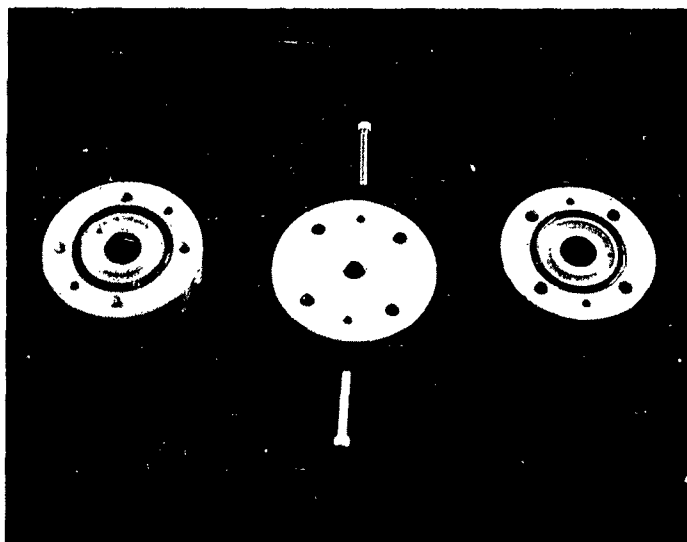
Fig. 3 Installed Orifice

Fig. 4 Orifice and Flange Assembly

pressures were evaluated at time increments. For unsteady flows, the differential pressure head across the nozzle recorded the velocity head and a small inertial head ($\Delta h = \frac{V^2}{2g} + 0.18 \frac{a}{g}$, Ref. 1). The inertial head correction was determined approximately from a potential flow net. Using this correction in a trial and error solution gave values of the instantaneous velocity. The acceleration was calculated by taking time incremental differences of the velocity. For steady flow, of course, the inertial head correction is zero. In all tests two pressure cells were arranged in parallel across the 12 diameter conduit test length to measure the instantaneous head drop. This duplication provided a check on the reliability of the transducing and recording system.

Fig. 5 shows the results of a typical test run with a variable acceleration. The fluid flowing at some low steady velocity, is subjected to a sudden impulse causing a sharp increase in acceleration. The velocity and head drop increase rapidly and continue to rise while the acceleration passes a peak and then falls off. In this example, the accelerated portion of the test run was completed in a two-second interval. From such data, values of K_a versus $\frac{aL}{V^2}$ were obtained for successive time intervals throughout the test.

From steady flow data, values of K_s versus Reynolds number were computed.

IV   RESULTS AND CONCLUSIONS

A. Discussion of Results

The experimental relations for the steady flow resistance coefficients are given in Table I.

Table I

Area Ratio (m^2)	Relation for K_s
1.00	$\frac{1}{\sqrt{K_s}} = 0.59 \log_{10} (R \sqrt{K_s}) - 0.54$
0.70	$K_s = 0.91$
0.50	$K_s = 3.68$

For the clear conduit ($m^2 = 1.00$) the pipe friction factor, f, was found equal to that given by the following equation for smooth pipes:

$$\frac{1}{\sqrt{f}} = 2.0 \log_{10} (F \sqrt{f}) - 0.8 \qquad\qquad (Eq. 23)$$

Substituting $K_s = f \frac{L}{D}$ with $\frac{L}{D} = 12$ for the test length used gives the equation in Table I. With either of the two orifices, the steady flow resistance is dependent primarily on the jet expansion process so that the resistance coefficient is essentially constant, independent of Reynolds number. The tabulated
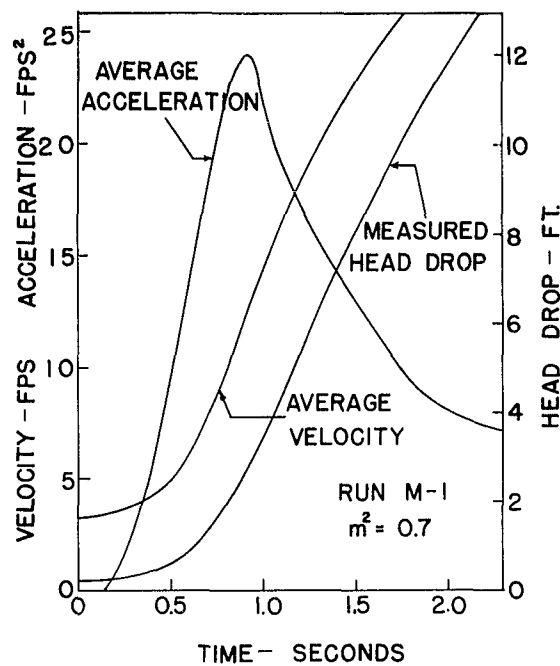
- 12 -
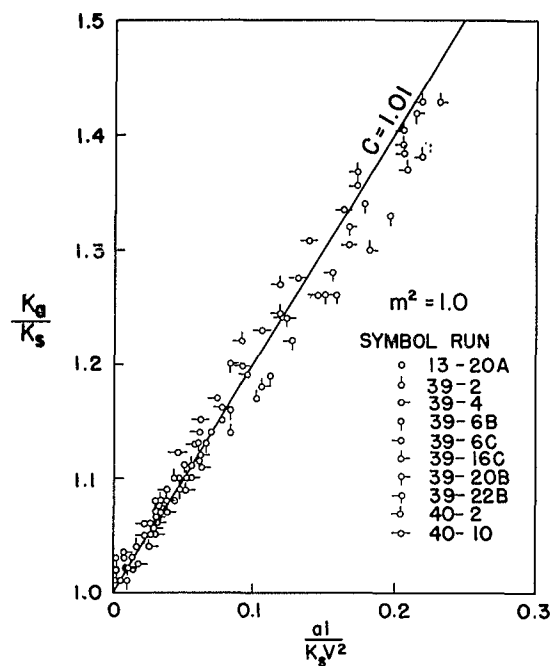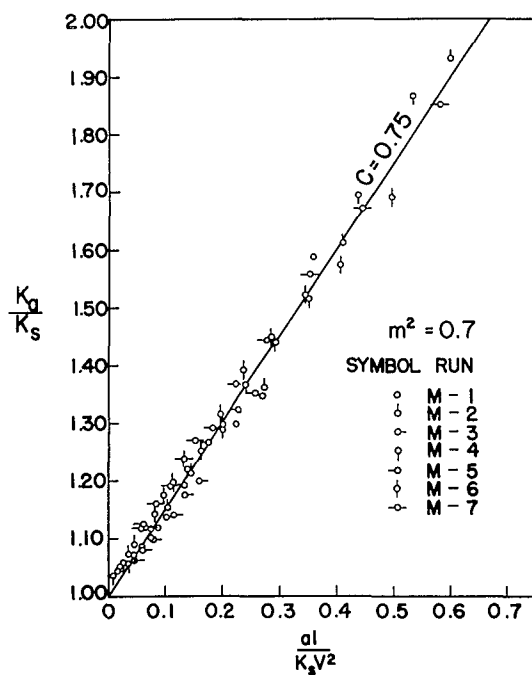
FIG. 5: TYPICAL TEST CURVES

FIG. 6: SMOOTH CONDUIT

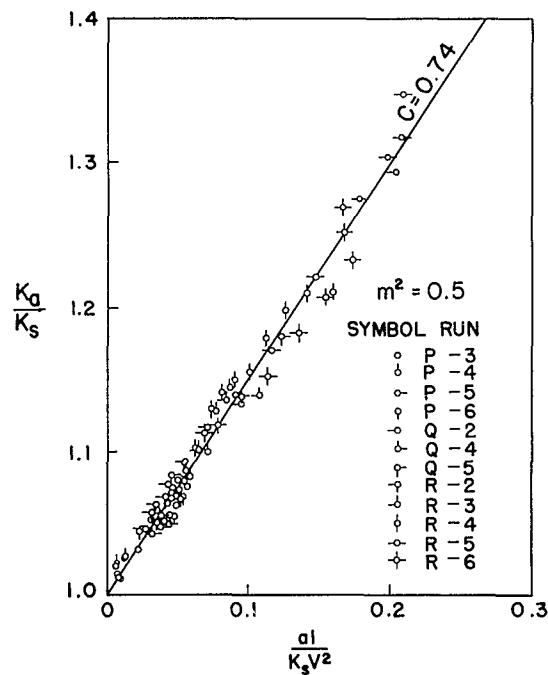FIG. 7: ORIFICE AREA RATIO = 0.7

FIG. 8: ORIFICE AREA RATIO = 0.5

FIG. 5 - 8: ACCELERATED FLOW TEST RESULTS

values are the slopes of mean lines drawn through the experimental points of steady flow head drop plotted versus $\frac{V^2}{2g}$.

Using these expressions for K_s, values of $\frac{K_a}{K_s}$ versus $\frac{aL}{K_s V^2}$ were obtained as shown in Figs. 6, 7 and 8. In the case of $m^2 = 1.00$, K_s at the instantaneous Reynolds number of the particular K_a values was used.

In each of the three diagrams of Figs. 6, 7 and 8, a mean straight line is drawn through the experimental data. While the individual points deviate considerably from this line, they can all be banded approximately by parallel straight lines. No other definite trends are indicated by any test run or combination of runs. The scatter is due in large measure to the extreme sensitivity of the ratios plotted to small errors. Of course these lines pass through the point ($\frac{K_a}{K_s} = 1.00$, $\frac{aL}{K_s V^2} = 0$) which any line or curve representing the physical process must do. Consequently, the slope of the line is in each case the value of the coefficient $2c$ in Eq. 21.

Table II gives the values of c, c_1 and c_2 for the three cases. For the clear tube, $c_1 = 1.00$ by definition. As previously noted, c_1 values for the orifices can be determined by numerical integration from a flow net of the jet profile. This profile is known only approximately and in addition is assumed to be essentially constant over the range of velocity and acceleration of the tests. Therefore, while the tabulated values of c_1 are the correct order of magnitude, they are accurate only to within an estimated $\pm 10\%$. The coefficients c_2 were obtained as the differences $(c - c_1)$.

<div align="center">Table II</div>

Area Ratio (m^2)	c	c_1	c_2
1.00	1.01	1.00	+0.01
0.70	0.75	1.15	-0.4
0.50	0.74	1.30	-0.6

Referring to Eq. 20, K_a is a function of Reynolds number as well as the acceleration parameter $\frac{aL}{V^2}$ unless both K_s and $c = c_1 + c_2$ are constants. For orifices K_s and c are in fact constants, so K_a is independent of Reynolds number. However, for the clear tube ($m^2 = 1.00$) the relation in Table I shows K_s to be a function of Reynolds number, while the relations derived from Schonfeld's theory

$$c_1 = 1.00$$

$$c_2 = \frac{0.91}{\left(\frac{1}{\sqrt{f}} + 0.87\right)^2}$$

<div align="center">- 14 -</div>

predicts c_2 and hence c to be also dependent on R. The variation in c is small, however, and the use of a single mean line to represent the data in Fig. 6 is justified as can be shown by numerical example. Using Schonfeld's theory over the range of Reynolds number investigated, c varies only between 1.010 and 1.015. Hence, in Fig 6. a single straight line with a slope indicating a constant c ≈ 1.01 is a satisfactory approximation.

This value of c = 1.01 makes c_2 positive for the clear conduit and implies a slight increase in pipe flow frictional resistance with acceleration. Expressing the sum of the frictional resistance (from Eq. 22) as a percentage of the steady state value gives

$$\frac{K_f + K}{K_s} = 1 + c_2 \frac{2aL}{K_s V^2} \qquad \text{(Eq. 24)}$$

With c_2 = +0.01, the increase would be less than one percent for $\frac{aL}{K_s V^2}$ less than 0.5. On the other hand, this small value of c_2 could mean very appreciable percentage increases for extreme cases with high values of $\frac{aL}{K_s V^2}$. Actually, the data in Fig. 6 could be represented just as well by a mean line falling slightly below the line shown and hence be in conformance with the previously reported conclusion of an inappreciable effect of acceleration (Refs. 1 and 2).

For both orifices, c_2 is definitely negative implying less frictional resistance at a given flow velocity with acceleration than without. Furthermore, the reduction becomes appreciable. Referring again to Eq. 24 for the frictional resistance as a percentage, it is seen that a negative c_2 also implies that the frictional resistance for a given velocity decreases with increasing acceleration. Thus for the 0.70 area ratio orifice, the resistance would be 60% of the equivalent steady state value for $\frac{aL}{K_s V^2}$ less than 0.5. At the highest values of $\frac{aL}{K_s V^2}$ of the tests (Fig. 7), the resistance is only about 50% of the equivalent steady flow resistance. For the 0.50 area ratio, the reduction is to 40% when $\frac{aL}{K_s V^2} = 0.5$.

It should be noted that the errors probable or possible in the determinations of c and c_1 would not alter these conclusions. The total spread of the plotted data in Figs. 7 and 8 corresponds to only a few percent variation in c, while c_1 must be no less than unity in any case. The combination of the extreme limits would not make c_2 positive for either of these examples.

It should be noted also that while c_1 is taken as a constant independent of Reynolds number and acceleration, this may be only approximately true. In which case c_2 will decrease as c_1 increases and vice versa. However, this would still not alter the previous conclusions.

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B. Summary of Conclusions

In summary, these experiments show that the pressure drop and resistance data for accelerated flow through a smooth tube and through orifices in a tube can be represented as functions of the acceleration parameter $\frac{aL}{V^2}$ and in particular the following two equations apply:

$$K_a = K_s + c\,\frac{2aL}{V^2}$$

$$K_f + K = K_s + c_2\,\frac{2aL}{V^2}$$

where for any particular conduit geometry, c is essentially constant and positive, and c_2 is essentially constant with a zero or a slightly positive value for a smooth tube and with negative values for orifices in the tube.

More specifically, it is found for orifices that

1. The coefficient of head drop, K_a, is independent of Reynolds number and dependent on the parameter $\frac{aL}{V^2}$.

2. The frictional resistance for a given instantaneous velocity of accelerated flow through an orifice in a tube is appreciably less than for steady flow at the same velocity.

3. The frictional resistance for a given instantaneous velocity of accelerated flow through an orifice in a tube decreases with increasing acceleration.

Also, a reanalysis of previous measurements with a smooth tube shows that

4. The coefficient of head drop, K_a, is a function of Reynolds number as well as the parameter $\frac{aL}{V^2}$.

5. The frictional resistance for a given instantaneous velocity of accelerated flow through a uniform diameter smooth tube is equal to, or possibly slightly greater than for steady flow at the same velocity.

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